The effect of wind on drag when riding a bike

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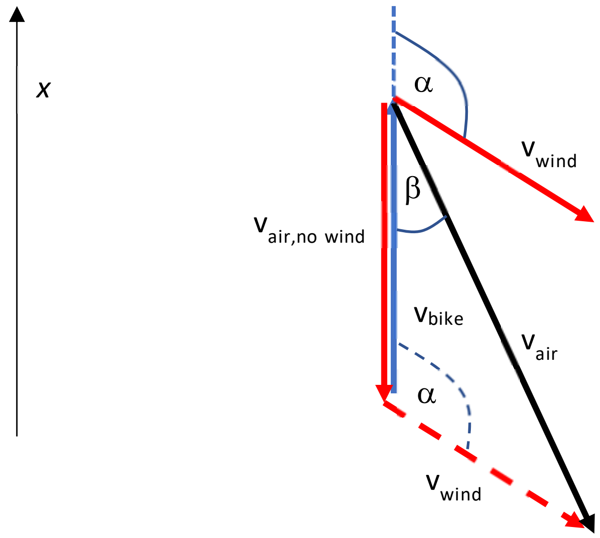
Have you ever had the feeling that wind is blowing in your face in both directions on your bike trip? Consideration of the drag shows that this is now only a feeling, but a physical reality.



Let us assume that you are riding a bike at a speed of v_{bike}. Consequently, air is moving in the opposite direction with the same speed ($v_{air,no\ wind}$). The wind blows at an angle of α with a speed of v_{wind}. The total velocity of the air relative to the bike is v_{air}. According to the law of cosines:

$$v_{air} = \sqrt{v_{bike}^2 + v_{wind}^2 - 2\, v_{bike}\, v_{wind} \cos\alpha} \tag{1}$$

The drag, denoted by F_d, is proportional to v_{air}^2:

$$F_d \sim v_{air}^2 \tag{2}$$

Drag is exerted in the direction of v_{air}. The component of drag parallel to the biker's velocity is:

$$F_{drag,\rightleftarrows} = F_d \cos\beta \tag{3}$$

According to the law of sines:

$$\frac{v_{wind}}{\sin\beta} = \frac{v_{air}}{\sin\alpha} \Rightarrow \sin\beta = \frac{v_{wind}\sin\alpha}{v_{air}} \tag{4}$$

Therefore:

$$F_{drag,\rightleftarrows} = F_d \cos\beta = F_d\sqrt{1-\sin^2\beta} = F_d\sqrt{1-\left(\frac{v_{wind}\sin\alpha}{v_{air}}\right)^2} \quad (5)$$

Putting all of these together:

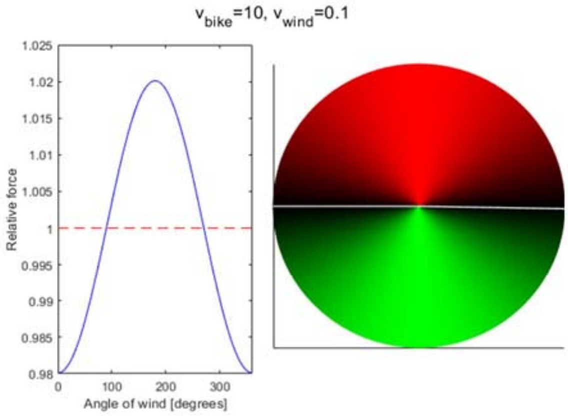
$$\begin{aligned} F_{drag,\rightleftarrows} &= F_d \cos\beta \sim v_{air}{}^2 \cos\beta = \\ &= v_{air}{}^2 \sqrt{1-\left(\frac{v_{wind}\sin\alpha}{v_{air}}\right)^2} = \\ &= \left(v_{bike}{}^2 + v_{wind}{}^2 - 2\, v_{bike}\, v_{wind}\cos\alpha\right)\sqrt{1-\left(\frac{v_{wind}\sin\alpha}{\sqrt{v_{bike}{}^2 + v_{wind}{}^2 - 2\, v_{bike}\, v_{wind}\cos\alpha}}\right)^2} \end{aligned} \quad (6)$$

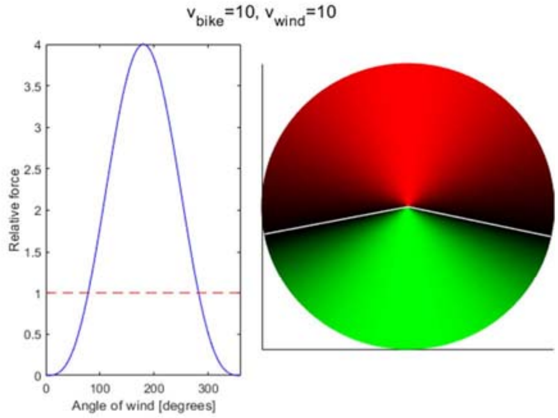
The relative magnitude of the drag parallel to the biker's velocity compared to the drag without wind is

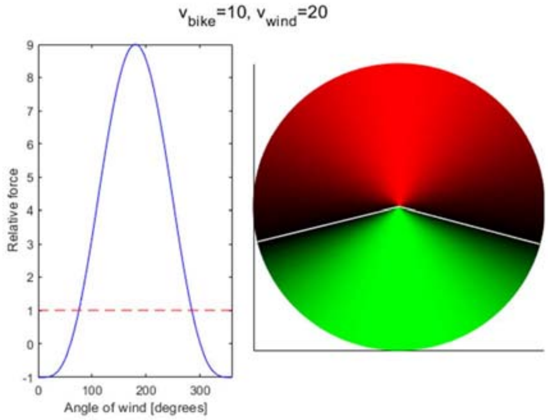
$$\frac{F_{drag,\rightleftarrows}}{F_{drag,no\ wind}} = \frac{\left(v_{bike}{}^2 + v_{wind}{}^2 - 2\, v_{bike}\, v_{wind}\cos\alpha\right)\sqrt{1-\left(\frac{v_{wind}\sin\alpha}{\sqrt{v_{bike}{}^2 + v_{wind}{}^2 - 2\, v_{bike}\, v_{wind}\cos\alpha}}\right)^2}}{v_{bike}{}^2} \quad (7)$$

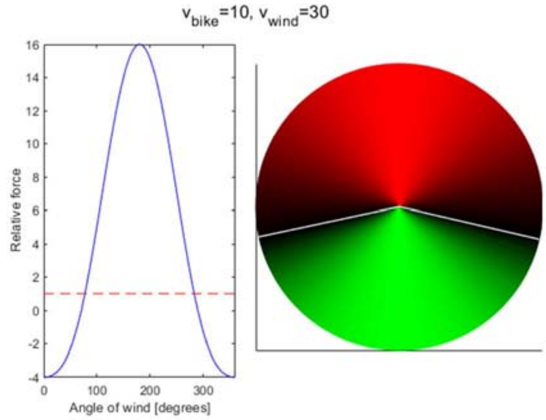
The direction of the F_{drag} (whether it is helping or blocking you) can be determined from the angle β. Below you can see a couple of charts at different speeds of the bike and the wind.

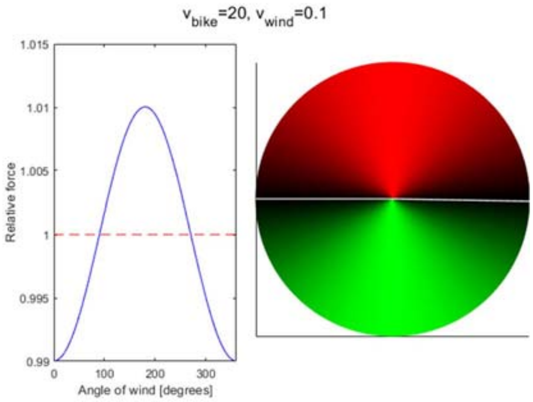
Relative force is 1 if you have to exert the same amount of force as in the absence of wind, it is more than 1 if you have to work harder, it is negative if wind actually propels you. Colored circles on the right: red and green correspond to relative forces of larger than 1 and less than 1, respectively. The angles around the circle correspond to the direction of wind. Upward and downward directions correspond to headwind and tailwind, respectively. The white lines separate the angle ranges in which the relative force is larger than one and smaller than one. As you can see, except for the weakest winds, wind is against the biker in more directions than it is helping the biker. Conclusions are the same, but more general than at https://alandix.com/blog/tag/cycling/

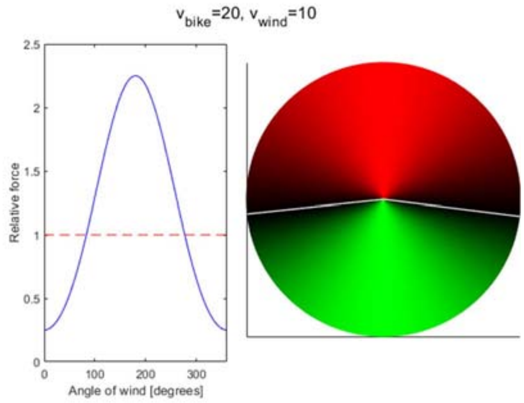

v_{bike}=10, v_{wind}=0.1
Relative force
Angle of wind [degrees]

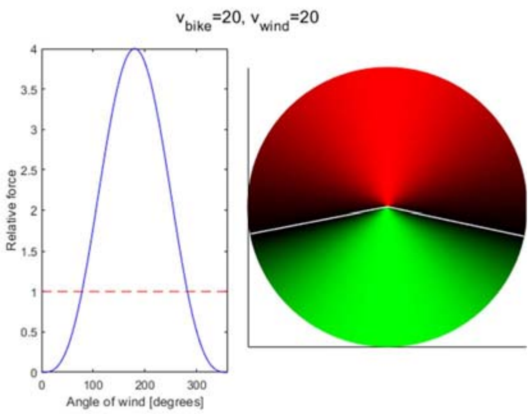

v_{bike}=10, v_{wind}=10
Relative force
Angle of wind [degrees]

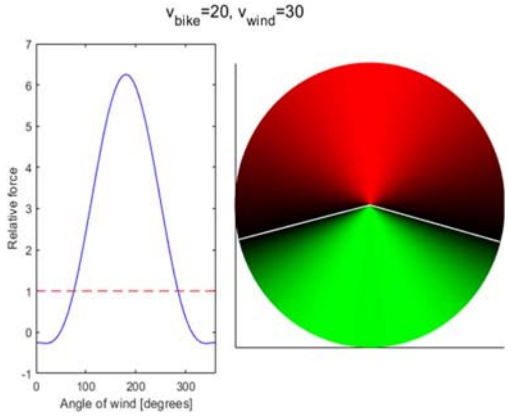

v_{bike}=10, v_{wind}=20
Relative force
Angle of wind [degrees]

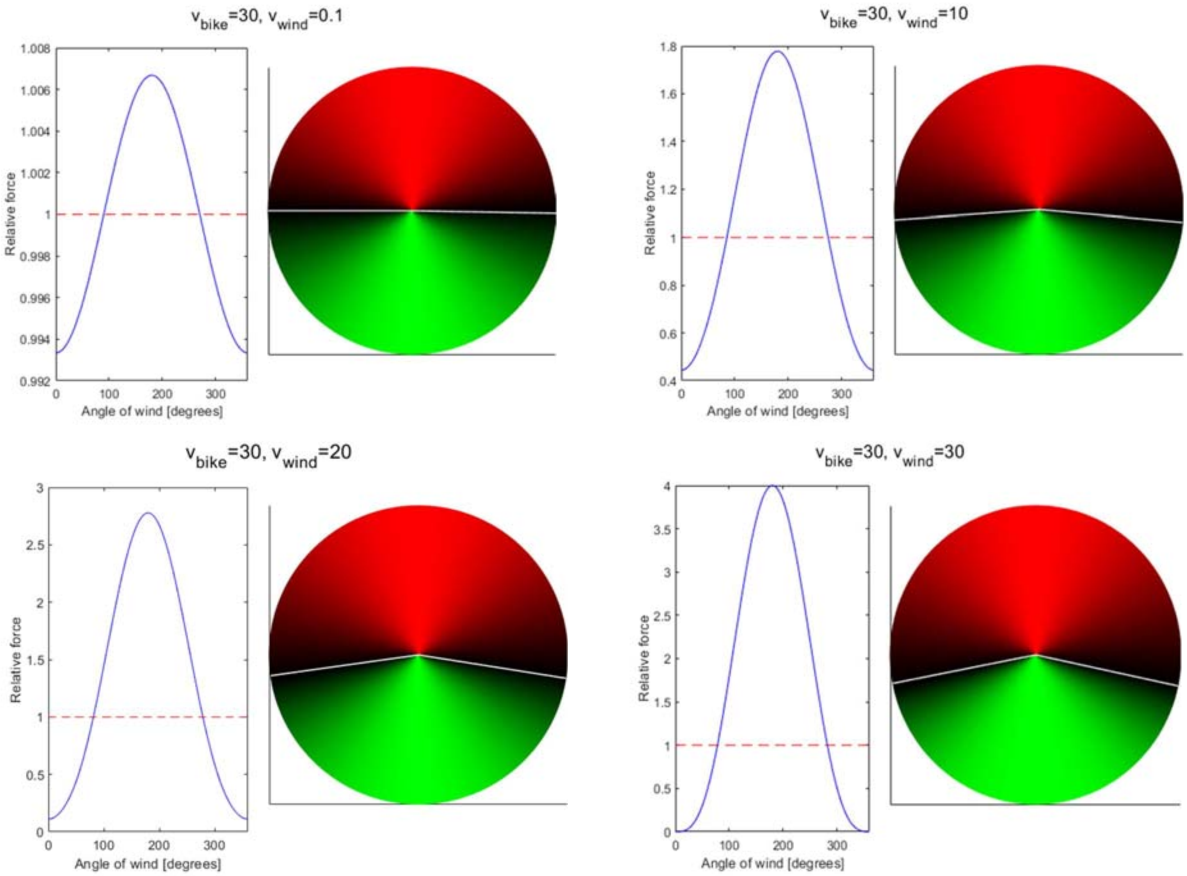

v_{bike}=10, v_{wind}=30
Relative force
Angle of wind [degrees]


v_{bike}=20, v_{wind}=0.1
Relative force
Angle of wind [degrees]


v_{bike}=20, v_{wind}=10
Relative force
Angle of wind [degrees]


v_{bike}=20, v_{wind}=20
Relative force
Angle of wind [degrees]


v_{bike}=20, v_{wind}=30
Relative force
Angle of wind [degrees]



The Matlab programs performing the calculations and making the graphs:

```
function [Frel,anglesDegree]=calculateDragInWindAllAngles(vbike,vwind)
anglesDegree=0:1:359;
anglesRad=anglesDegree*pi/180;
[Frel,cosbeta]=calculateDragInWind(vbike,vwind,anglesRad);
direction=ones(1,360);
if vwind>vbike
    direction(1)=-1;
end
signChange=[false, false, diff(sign(diff(cosbeta)))~=0];
signChangeAtLowCosBeta=signChange & cosbeta<0.1;
direction(signChangeAtLowCosBeta)=-1;
direction=cumprod(direction);
FrelCorrected=Frel.*direction;
figure;
subplot(1,2,1);
plot(anglesDegree,FrelCorrected,'b');
hold on;
plot(0:359,ones(1,360),'r--');
xlabel('Angle of wind [degrees]');
ylabel('Relative force');
set(gca,'xlim',[0 360]);
lengthOfColorScale=256;
forceScale=linspace(min(FrelCorrected),max(FrelCorrected),lengthOfColorScal
e);
posOfOne=round((1-min(FrelCorrected))/(max(FrelCorrected)-
min(FrelCorrected))*lengthOfColorScale);
colorMap=zeros(lengthOfColorScale,3);
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```
colorMap(posOfOne:end,1)=linspace(0,1,lengthOfColorScale-posOfOne+1)';
colorMap(1:posOfOne,2)=linspace(1,0,posOfOne);
axes('parent',gcf,'position',[0.5 0.05 0.5 0.9]);
FrelExtended=[FrelCorrected,FrelCorrected(1)];
anglesRadExtended=[anglesRad,anglesRad(1)];
for i=1:360
    colorIndex1=find(abs(FrelExtended(i)-
forceScale)==min(abs(FrelExtended(i)-forceScale)),1);
    colorIndex2=find(abs(FrelExtended(i+1)-
forceScale)==min(abs(FrelExtended(i+1)-forceScale)),1);
    colorIndex=round(mean(colorIndex1,colorIndex2));
    patch([0 sin(anglesRadExtended(i)) sin(anglesRadExtended(i+1)) 0],-[0
cos(anglesRadExtended(i)) cos(anglesRadExtended(i+1))
0],colorMap(colorIndex,:),'edgecolor',colorMap(colorIndex,:));
end
oneCrossing=find(diff(sign(FrelCorrected-1))~=0 &
abs(sign(FrelCorrected(1:end-1)-1))==1);
for i=1:2
    line([0 sin(anglesRad(oneCrossing(i)))],-[0
cos(anglesRad(oneCrossing(i)))],'color','white','linewidth',1);
end
set(gca,'ytick',[],'xtick',[],'box','off','dataaspectratio',[1 1
1],'plotboxaspectratio',[1 1 1]);
sgtitle(['v_b_i_k_e=',num2str(vbike),', v_w_i_n_d=',num2str(vwind)]);



function [Frel,cosbeta]=calculateDragInWind(vbike,vwind,anglesRad)
vair=sqrt(vbike^2+vwind^2-2*vbike*vwind*cos(anglesRad));
cosbeta=sqrt(1-(vwind*sin(anglesRad)./vair).^2);
cosbeta(isnan(cosbeta))=0;
Frel=vair.^2.*sqrt(1-(vwind*sin(anglesRad)./vair).^2)/vbike^2;
Frel(isnan(Frel))=0;
```